FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

Roll Number

STATISTICS

TIME ALLOWED: THREE HOURS	**(PART-I MCQs)**	**30 MINUTES**	**MAXIMUM MARKS: 20**
	(PART-II)	**2 HOURS & 30 MINUTES**	**MAXIMUM MARKS: 80**

NOTE: (i) **First** attempt **PART-I (MCQs)** on separate Answer Sheet which shall be taken back after 30 minutes.
(ii) **Overwriting/cutting of the options/answers will not be given credit.**
(iii) **Statistical Tables will be provided if required.**
(iv) **Use of Scientific calculator is allowed.**

(PART-I MCQs) (COMPULSORY)

Q.1. Select the best option/answer and fill in the appropriate box on the Answer Sheet. **(1 x 20=20)**

(i) The mean of X, following a Binomial distribution with parameter n and p is _____ variance of x.

(a) Equal to the (b) Less than the (c) Greater than the
(d) Equal to the square root of the (e) None of these

(ii) $(A \cap B) \cup (A \cap B') =$

(a) A (b) B (c) A' (d) B' (e) None of these

(iii) Four candidates are seeking a vacancy on a college board. If A is twice as likely to be elected as B, and B and C are given about the same chance of being elected, while C is twice as likely as D, what are the chances that C will be elected?

(a) $\frac{1}{2}$ (b) $\frac{2}{9}$ (c) $\frac{1}{3}$ (d) $\frac{4}{9}$ (e) None of these

(iv) For married couple in a certain locality the probability that the husband will watch a specific TV program is 0.21, the probability that the wife will watch that TV program is 0.28 and the probability that both husband and wife will watch that TV program is 0.15. What is the probability that at least one of them will watch that TV program?

(a) 0.49 (b) 0.64 (c) 0.34 (d) 0.36 (e) None of these

(v) The value of k that will make the function, $f(x, y) = kxy$ for x = 1, 2, 3 and y = 1, 2, 3 a joint probability distribution is:

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{36}$ (e) None of these

(vi) If the joint probability density function of X and Y is given by $f(x, y) = 2$ for x>0 and y>0 and zero elsewhere, then $P(x < 1/2, y < 1/2) =$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$ (e) None of these

(vii) If V(x)=19 then V(2x –5) =

(a) 19 (b) 38 (c) 33 (d) 76 (e) None of these

(viii) Assume that the fitted regression between x and y is, $y = \beta_1 + \beta_2 x$ and the regression fitted between z and w is $z = \beta_3 + \beta_4 x$. Given that z = 3y and w = 2x, then:

(a) $\beta_4 = \beta_2$ (b) $\beta_4 = (3/2)\beta_2$
(c) $\beta_4 = (2/3)\beta_2$ (d) $\beta_4 = 4\beta_2$ (e) None of these

STATISTICS

(ix) While applying analysis of variance to test the equality of means, three conditions namely, independence, normality and ______ must hold,

(a) Consistency (b) Unbiasedness

(c) Homogeneity of population variances (d) Efficient estimators (e) None of these

(x) In traditional sampling theory the finite population correction factor is denoted by

(a) (N – n)/(N – 1) (b) (N – n)/N

(c) N/(N – 1) (d) n/(N – 1) (e) None of these

(xi) A random sample of size n is drawn from a population following exponential distribution with probability density function, $f(x)=\frac{1}{\lambda}e^{-x/\lambda}$, for x>0. Then the maximum likelihood estimator of λ is given by

(a) $\bar{x}$ (b) $1/\bar{x}$ (c) $\sum_{i=1}^{n} x_i$ (d) $\sum_{i=1}^{n} x_i^2$ (e) None of these

(xii) An estimator $\hat{\theta}$ is said to be consistent if

(a) $E(\hat{\theta})=\theta$ (b) $E(\hat{\theta})=V(\hat{\theta})$

(c) $V(\hat{\theta})\to 0$ as $n\to\infty$ (d) $V(\hat{\theta})=[E(\hat{\theta})]^2$ (e) None of these

(xiii) If b is constant and the moment generating function of x is $M_x(t)$ then $M_{x+b}(t)$ =

(a) $M_x(t)$ (b) $M_x(bt)$ (c) $M_x(t)+b$ (d) $e^{bt}M_x(t)$ (e) None of these

(xiv) If the random variable x is distributed normally, N(105,36) then w = (x – 105)/6 will follow a normal distribution,

(a) N (105, 1) (b) N (0, 1) (c) N (105, 6) (d) N (105, 36) (e) None of these

(xv) If the coefficient of correlation between two variables x and y is given by r, then the coefficient of correlation between z = ax+b and w=cy+d will be equal to

(a) (ac+bd)r (b) (acbd)r (c) r (d) (ac)r+bd (e) None of these

(xvi) Assuming x, y and z are three variables, then using the usual notations, the partial correlation coefficient, $R_{xy.z}$ is given by

(a) $(r_{xy}-r_{xz})\big/\sqrt{(1-r_{xy}^2)}$ (b) $(r_{xy}-r_{xz}r_{yz})\big/\sqrt{1-r_{xy}^2}$

(c) $(r_{xy}-r_{xz})\big/\left[\left(\sqrt{1-r_{xy}^2}\right)\left(\sqrt{1-r_{yz}^2}\right)\right]$

(d) $(r_{xy}-r_{xz}r_{yz})\big/\left[\left(\sqrt{1-r_{xz}^2}\right)\left(\sqrt{1-r_{yz}^2}\right)\right]$ (e) None of these

(xvii) A stock may result in profit of \$1, loss of \$1 or breakeven (no gain no loss) with respective probabilities 0.4, 0.3 and 0.3 then the average profit will be

(a) \$1.0 (b) \$0.4 (c) \$0.25 (d) \$0.1 (e) None of these

(xviii) While expanding the moment generating function the coefficient of μ_r' is given by

(a) $t^r/r!$ (b) t^r/r (c) t^r (d) $r!t^r$ (e) None of these

(xix) Assume that x and y are two independent random variables then the V(xy) is equal to

(a) xy (b) zero (c) x/y (d) x+y (e) None of these

(xx) If A and B are two independent variables then the conditional probability $P(B|A)=$

(a) $P(A\cap B)$ (b) $P(A)$ (c) $P(B)$ (d) Zero (e) None of these

STATISTICS

PART-II

NOTE:(i)	**PART-II** is to be attempted on separate Answer Book.
(ii)	**Attempt ANY FIVE questions from PART-II. All questions carry EQUAL marks.**
(iii)	**Extra attempt of any question or any part of the attempted question will not be considered.**

Q.2. (a) Differentiate between independent, dependent and mutually exclusive events. Give one example for each type of event. **(06)**

(b) A shipment of 10 TV sets includes three that are defective. A store dealer purchases four TV sets randomly. Find: **(06)**

(i) Probability of getting exactly two defective TV sets;

(ii) Probability of getting at least one defective TV set.

(c) In a large city the probabilities that a family, selected randomly, has a black or coloured mobile phone set is 0.86 and 0.35, respectively. Further the probability that the family has both black and coloured mobile phone set is 0.29. A family from this city is selected randomly, what is the probability that the family possesses either or both types of mobile phones. **(04)**

Q.3. (a) A delicate surgical operation is quite successful and the probability of its failure is 0.005. What is the probability that among next 1000 patients, having this surgical operation, **(04 + 04)**

(i) Exactly five will not survive?

(ii) At least two will not survive?

(b) Let x, a random variable showing the number of calls arriving at a telephone exchange during a specific time period, follows a probability distribution given by $f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$ for x = 0, 1, 2, … and $\lambda > 0$. Determine moment generating function and find mean and variance of x. **(04 + 02 + 02)**

Q.4. (a) Assuming that a random variable x, representing the life of a specific type of tube light follows a normal distribution with probability density function,

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, where -\infty < x < \infty.$$

(i) Show that f(x) is a probability density function. **(04)**

(ii) Determine maximum likelihood estimators of μ and σ^2. **(06)**

(b) A manufacturer claims that at most 5 percent of the time a given product will sustain fewer than 1000 hours of operation before requiring service. Twenty products were selected randomly from the production line and tested. It was found that three of them required service before 1000 hours of operation. Comment on the manufacturer's claim. **(06)**

Q.5. A pharmaceutical company ABC recently launched a new medicine to provide an early recovery to severe headache patients. The company ABC has announced that their medicine named, NewMed provides, on average, early recovery than the existing medicine ExMed. To test their claim a study was conducted and patients with severe headache were administered these medicines on random basis. Following table shows the recovery times of 13 such patients.

ExMed	12	23	22	12	13	14	
NewMed	12	9	11	10	9	8	10

(i) Do the data provide sufficient evidence, at 5% level of significance, to accept the claim of ABC? **(06)**

(ii) Construct a 90% confidence interval for $\mu_{New\,Med} - \mu_{Ex\,Med}$, and comment on the result. **(06)**

(iii) Construct a 99% confidence interval for $\sigma^2_{New\,Med}$ and comment on the finding. **(04)**

STATISTICS

Q.6. **(a)** Considering the simple linear regression model, $y_i = \beta_1 + \beta_2 x_i + e_i$, for i = 1, 2, …, n, state assumptions and derive least square estimators of the β_1 and β_2. **(02 + 06)**

(b) Following table shows the income and saving of seven families residing at a specific locality.

Income (I)	9	11	13	15	17	19	21
Saving (S)	5	6	9	11	12	14	15

(i) Fit a regression model, $\hat{S}_i = \hat{\beta}_1 + \hat{\beta}_2 I_i$ for i = 1, 2, …, 7. **(05)**

(ii) Test the hypothesis $H_0 : \beta_2 = 1$ against $H_0 : \beta_2 < 1$ at 5% level of significance. **(03)**

Q.7. **(a)** Assume that a random sample of size n is drawn from a population of size N. the population is further assumed to have a mean μ and variance σ^2. Prove that, $V(\bar{y}) = \frac{\sigma^2}{n}\left(\frac{N-n}{N}\right)$. **(08)**

(b) Draw all possible samples of size 3, without replacement, from the population: 12, 9, 15, 9 and 21 and prove that $E(\bar{y}) = \mu$ **(08)**

Q.8. **(a)** A study was conducted to establish relationship between the nature of crime and educational facilities available. The study was based on 291 respondents and the number of respondents found involved in various types of crimes were recorded as given below. Data collected during the study is also given below: **(08)**

	Nature of Crime		
Education Level	Low	Medium	High
Low	17	22	47
Medium	12	15	22
High	32	21	14
Very High	45	33	11

Could it be concluded, at 1% level of significance, that there exists a significant association between the availability of education facility and nature of crime?

(b) A study was conducted to compare the lifespan of three types of batteries. Fifteen batteries, five of each type, were selected randomly from the production line and observed till they expired. Their lifespans, as recorded, are given below: **(08)**

Battery Type		
A	B	C
23	23	54
34	22	56
44	21	55
45	23	67
44	34	65

Test the hypothesis, $H_0 : \mu_A = \mu_B = \mu_C$ at 5% level of significance.

Q.9. Write short notes on the following topics: **(04 + 04 + 04 + 04=16)**

(a) Role of statistics in highlighting socio-economic problems of a society.

(b) Comparison and advantages of Stratified and Systematic sampling schemes.

(c) Partial and Multiple regression and correlations.

(d) Importance of hypothesis testing in real life situations.

****