FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

Roll Number

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: (i) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B.** All questions carry equal marks.
(ii) **Use of Scientific Calculator is allowed.**
(iii) **Extra attempt of any question or any part of the attempted question will not be considered.**

SECTION - A

Q.1. (a) Find the divergence and curl $\vec{f}$ If $\vec{f} = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + (x^2y + 3z^2)\hat{k}$ **(10)**

(b) Also find a function φ such that $\nabla\varphi = \vec{f}$ **(10)**

Q.2. (a) Find the volume $\iint_R xy\, dA$ where R is the region bounded by the line y = x – 1 and the parabola $y^2 = 2x + 6$. **(10)**

(b) Evaluate the following line intergral: **(10)**

$\int_C y^2dx + xdy \;\; where \;\; c = c_2$ is the line segment joining the points (-5, -3) to (0, 2), and c = c_2 is the arc of the parabola $x = 4 - y^2$.

Q.3. (a) Three forces P, Q and R act at a point parallel to the sides of a triangle ABC taken in the same order. Show that the magnitude of the resultant is **(10)**
$\sqrt{p^2 + Q^2 + R^2 - 2QR\cos A - 2RP\cos B - 2PQ\cos C}$

(b) A hemispherical shell rests on a rough inclined plane whose angle of friction is λ. Show that the inclination of the plane base to the horizontal cannot be greater than arcsin(**2 sin** λ) **(10)**

Q.4. (a) A uniform square lamina of side $2a$ rests in a vertical plane with two of its sides in contact with two smooth pegs distant b apart and in the same horizontal line. Show that if $\frac{\theta}{\sqrt{2}} < b < a$, a non symmetric position of equilibrium is possible in which $b(\sin\theta + \cos\theta) = a$ **(10)**

(b) Find the centre of mass of a semi circular lamina of radius a whose density varies as the square of the distance from the centre. **(10)**

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Q.5. (a) Evaluate the integral $\int_0^1 \int_{x^2}^{x} (x^2 + y^2) dy dx$ **(10)**

also show that the order of integration is immaterial.

(b) Find the directional derivative of the function at the point P along z – axis **(10)**

$f(x, y) = 4xz^3 - 3x^2 y^2 z, P = (2, -1, 2)$

SECTION – B

Q.6. (a) A particle is moving along the parabola $x^2 = 4ay$ with constant speed v. Determine the tangential and the normal components of its acceleration when it reaches the point whose abscissa is $\sqrt{5}a$ **(10)**

(b) Find the distance travelled and the velocity attained by a particle moving in a straight line at any time t, if it starts from rest at t =0 and is subject to an acceleration $t^2 + \sin t + e^t$ **(10)**

Q.7. (a) A particle moves in the xy – plane under the influence of a force field which is parallel to the axis of y and varies as the distance from x – axis. Show that, if the force is repulsive, the path of the particle supposed not straight and then **(10)**

y = a cosh nx + a sinh nx

where a and b are constants.

(b) Discuss the motion of a particle moving in a straight line with an acceleration x^3, where x is the distance of the particle from a fixed point O on the line, if it starts at t = 0 from a point x = c with the velocity $c^2/\sqrt{2}$. **(10)**

Q.8. (a) A battleship is steaming ahead with speed V and a gun is mounted on the battleship so as to point straight backwards and is set at angle of elevation α. If v_0 is the speed of projection (relative to the gun) show that the range is $\frac{2v_0}{g} \sin\alpha (v_0 \cos\alpha - V)$ **(10)**

(b) Show that the law of force towards the pole of a particle describing the survey $r^n = a^n \cos n\theta$ is given by $f = \frac{(n+1)h^2 a^{2n}}{r^{2n+3}}$ where h is a constant. **(10)**

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