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FEDERAL PUBLIC SERVICE COMMISSION COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2009

S.No. R.No.

TIME ALLOWED: 3 HOURS

## MAXIMUM MARKS:100

NOTE:	<ul> <li>(i) Attempt FIVE questions in all by selecting at least THREE questions from SECTION-A and TWO question from SECTION-B. All questions carry EQUAL marks.</li> <li>(ii) Use of Scientific Calculator is allowed.</li> </ul>
	<u>SECTION – A</u>
<b>Q.1.</b> (a)	Prove that the set $S_n$ of all permutations on a set X of n elements is a group under the operation 'o' of composition of permutations. Will ( $S_n$ , o) be an abelian group? How do we call this group? (10)
(b)	If G is a group, N a normal subgroup of G, then show that the set G/N of right cosets of N in G is also a group. How we call this group? Also, if G is finite then show that $f(G) = f(G)$
	$o\left(\frac{G}{N}\right) = \frac{o(G)}{o(N)}.$ (10)
<b>Q.2.</b> (a)	Let cp be a homomorphism of a group G onto another group H with kernel K. Prove that $G/K$ is isomorphic to H, that is $G/K \approx H$ . (10)
(b)	Let $Z_n$ be the set of the congruence classes modulo n, that is, $Z_n = \{[0], [1], [2], \dots, [n-1]\}$ Define the two binary operations on $Z_n$ under which it is a ring. Prove that the ring $Z_n$ is an integral domain $\Leftrightarrow$ n is a prime number. (10)
<b>Q.3.</b> (a)	Let $T : R^3 \rightarrow R^3$ be the linear mapping defined by: T(x,y,z) = (x+2y-z, y+z, x+2y-z) Verify that Rank (T) + Nullity (T) = dim D(T) Also find a basis for each Rank (T) and Nullity (T) (10)
(b)	Also find a basis for each Kank (1) and Runny (1)(10)If U and W are finite – dimensional subspaces of a vector space V over a field F then prove that $\dim(U+W) + \dim(U \cap W) = \dim U + \dim W$ (10)
<b>Q.4.</b> (a)	Let H and K be two subgroups of a group G. Prove that HK is a subgroup of $G \Leftrightarrow HK = KH$ . (10)
(b)	Let $v_1, v_2, \ldots, v_n$ be non-zero eigen vectors of an operator T:V $\rightarrow$ V belonging to distinct eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$ . Show that the vectors $v_1, v_2, \ldots, v_n$ are linearly independent.

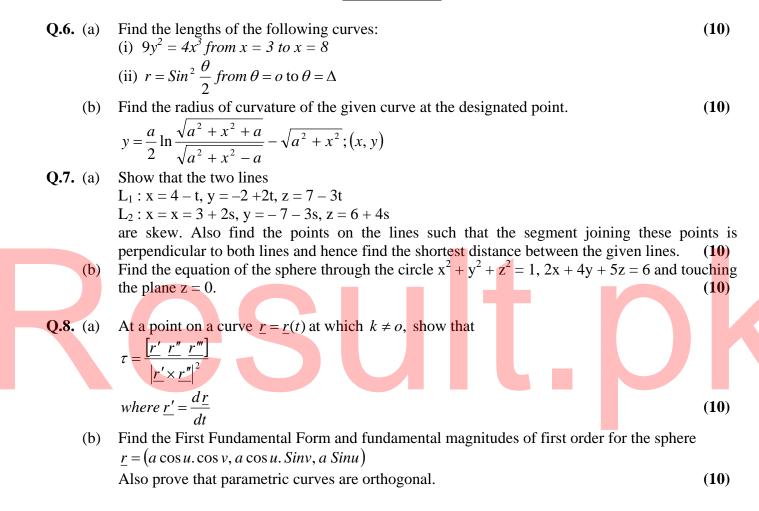
(10)

## **PURE MATHEMATICS, PAPER-I**

- **Q.5.** (a) Let V be the vector space of n-square matrices over the field IR. Let U and W be the subspaces of symmetric and antisymmetric matrices, respectively. Show that  $V = U \oplus W$ . (10)
  - (b) Diagonalize the following matrix:

$$M = \begin{bmatrix} -4 & -4 & -8 \\ 4 & 6 & 4 \\ 6 & 4 & 10 \end{bmatrix}$$

## **SECTION – B**



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