FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2009

PURE MATHEMATICS, PAPER-I

S.No.	
R.No.	

TIME ALLOWED: 3 HOURS — **MAXIMUM MARKS:100**

NOTE:
(i) Attempt **FIVE** questions in all by selecting at least **THREE** questions from **SECTION–A** and **TWO** question from **SECTION–B**. All questions carry **EQUAL** marks.
(ii) Use of Scientific Calculator is allowed.

SECTION – A

Q.1. (a) Prove that the set S_n of all permutations on a set X of n elements is a group under the operation 'o' of composition of permutations. Will (S_n, o) be an abelian group? How do we call this group? **(10)**

(b) If G is a group, N a normal subgroup of G, then show that the set G/N of right cosets of N in G is also a group. How we call this group? Also, if G is finite then show that

$$o\left(G/N\right) = \frac{o(G)}{o(N)}.$$ **(10)**

Q.2. (a) Let cp be a homomorphism of a group G onto another group H with kernel K. Prove that G/K is isomorphic to H, that is G/K ≈ H. **(10)**

(b) Let Z_n be the set of the congruence classes modulo n, that is,
$Z_n = \{[0], [1], [2], \ldots\ldots [n-1]\}$
Define the two binary operations on Z_n under which it is a ring. Prove that the ring Z_n is an integral domain $\Leftrightarrow$n is a prime number. **(10)**

Q.3. (a) Let $T : R^3 \rightarrow R^3$ be the linear mapping defined by:
$T(x,y,z) = (x+2y - z, y + z, x+2y - z)$
Verify that
Rank (T) + Nullity (T) = dim D(T)
Also find a basis for each Rank (T) and Nullity (T) **(10)**

(b) If U and W are finite – dimensional subspaces of a vector space V over a field F then prove that
$\dim(U+W) + \dim(U\cap W) = \dim U + \dim W$ **(10)**

Q.4. (a) Let H and K be two subgroups of a group G. Prove that HK is a subgroup of G $\Leftrightarrow$ HK = KH. **(10)**

(b) Let $v_1, v_2, \ldots\ldots, v_n$ be non-zero eigen vectors of an operator $T:V\rightarrow V$ belonging to distinct eigen values $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$. Show that the vectors $v_1, v_2, \ldots\ldots, v_n$ are linearly independent. **(10)**

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Q.5. (a) Let V be the vector space of n-square matrices over the field IR. Let U and W be the subspaces of symmetric and antisymmetric matrices, respectively. Show that V = U ⊕ W. **(10)**

(b) Diagonalize the following matrix:

$$M = \begin{bmatrix} -4 & -4 & -8 \\ 4 & 6 & 4 \\ 6 & 4 & 10 \end{bmatrix}$$

SECTION – B

Q.6. (a) Find the lengths of the following curves: **(10)**

(i) $9y^2 = 4x^3$ *from* $x = 3$ *to* $x = 8$

(ii) $r = Sin^2 \dfrac{\theta}{2}$ *from* $\theta = o$ to $\theta = \Delta$

(b) Find the radius of curvature of the given curve at the designated point. **(10)**

$$y = \frac{a}{2} \ln \frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a} - \sqrt{a^2 + x^2}; (x, y)$$

Q.7. (a) Show that the two lines

L_1 : x = 4 – t, y = –2 +2t, z = 7 – 3t

L_2 : x = x = 3 + 2s, y = – 7 – 3s, z = 6 + 4s

are skew. Also find the points on the lines such that the segment joining these points is perpendicular to both lines and hence find the shortest distance between the given lines. **(10)**

(b) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 1$, $2x + 4y + 5z = 6$ and touching the plane z = 0. **(10)**

Q.8. (a) At a point on a curve $\underline{r} = \underline{r}(t)$ at which $k \neq o$, show that

$$\tau = \frac{[\underline{r}' \ \underline{r}'' \ \underline{r}''']}{|\underline{r}' \times \underline{r}''|^2}$$

where $\underline{r}' = \dfrac{d\underline{r}}{dt}$ **(10)**

(b) Find the First Fundamental Form and fundamental magnitudes of first order for the sphere

$\underline{r} = (a \cos u . \cos v, a \cos u . Sin v, a\ Sin u)$

Also prove that parametric curves are orthogonal. **(10)**

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